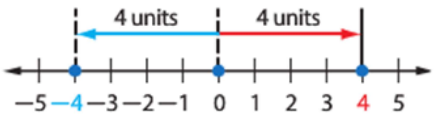
Lesson 2-7: Solving Equations Involving Absolute Value

WHAT IS ABSOLUTE VALUE?

In operations with integers, **absolute value** is important. This is the value of a number when we ignore the positive or negative sign. Another way to think of absolute value is that it is a number's distance from zero on the number line, in either direction. The absolute value of any number *n* is written $|n|$ and is **always positive**.

absolute value notation

For example, let's look at the equation $|x| = 4$. This means that the distance between 0 and x on a number line is 4.



$|x| = 4$

$x = 4, -4$

Before we start working on solving absolute value equations, let's review how to evaluate absolute value expressions.

EXAMPLE 1

Evaluate $|m + 6| - 14$ if m = 4.

Plug in 4 for m: $= |\underline{4} + 6| - 14$

Do what's inside the absolute value: $= |\underline{10}| - 14$

Do the absolute value: $= \underline{10} - 14$

Simplify: $= \underline{-4}$

EXERCISE 1: Evaluate $23 - |3 - 4x|$ if x = 2.

$23 - |3 - 4 \cdot 2|$

$23 - |3 - 8|$

$23 - |-5|$

$23 - 5$

18

always do PEMDAS

KeyConcept Absolute Value Equations

Words	When solving equations that involve absolute values, there are two cases to consider.
	Case 1 The expression inside the absolute value symbol is positive or zero.
	Case 2 The expression inside the absolute value symbol is negative.
Symbols	For any real numbers a and b, if $\lvert a \rvert = b$ and $b \geq 0$, then $a = b$ or $a = -b$.
Example	$\lvert d \rvert = 10$, so $d = 10$ or $d = -10$.

because: $|10| = 10$ or $|-10| = 10$

EXAMPLE 2

Solve each equation. Then graph the solution set.

a. $|f + 5| = 17$ → what numbers have an absolute value of 17?

$|f + 5| = 17$ Original equation

$|17| = 17$ or $|-17| = 17$

Case 1 Case 2

Case 1

$f + 5 = 17$

$f + 5 - 5 = 17 - 5$ Subtract 5 from each side.

$f = 12$ Simplify.

ch: $|12 + 5| = 17$

$|17| = 17$

$17 = 17$ ✓

Case 2

$f + 5 = -17$

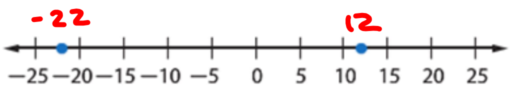
$f + 5 - 5 = -17 - 5$

$f = -22$

ch: $|-22 + 5| = 17$

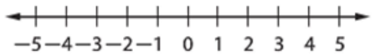
$|-17| = 17$

$17 = 17$ ✓



b. $|b - 1| = -3$

$|b - 1| = -3$ means the distance between b and 1 is -3. Since distance cannot be negative, the solution is the empty set $\varnothing$. → no solution

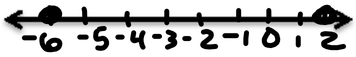


no solution means nothing on the number line!

Absolute values cannot equal negative numbers EVER.

EXERCISES Solve each equation. Then graph the solution set.

2. $|y + 2| = 4$



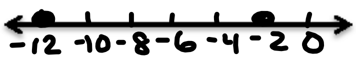
Case 1
$|4| = 4$
$y + 2 = 4$
$-2 \mid -2$
$y = 2$

Case 2
$|-4| = 4$
$y + 2 = -4$
$-2 \mid -2$
$y = -6$

3. $|3n - 4| = -1 \rightarrow \varnothing$

$\rightarrow$ blank

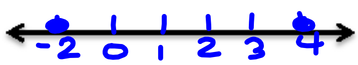
4. $|n + 7| = 5$



Case 1
$|5| = 5$
$n + 7 = 5$
$-7 \mid -7$
$n = -2$

Case 2
$|-5| = 5$
$n + 7 = -5$
$-7 \mid -7$
$n = -12$

5. $|3z - 3| = 9$



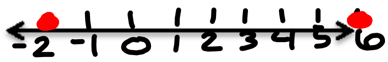
Case 1
$|9| = 9$
$3z - 3 = 9$
$+3 \quad +3$
$\frac{3z}{3} = \frac{12}{3}$
$z = 4$

Case 2
$|-9| = 9$
$3z - 3 = -9$
$+3 \quad +3$
$\frac{3z}{3} = \frac{-6}{3}$
$z = -2$

6. $|4n - 1| = -6 \rightarrow \varnothing$

blank

7. $|2t - 4| = 8$



Case 1
$|8| = 8$
$2t - \cancel{4} = 8$
$\cancel{+4} \mid +4$
$\frac{\cancel{2}t}{\cancel{2}} = \frac{12}{2}$
$t = 6$

Case 2
$|-8| = 8$
$2t - \cancel{4} = -8$
$\cancel{+4} \mid +4$
$\frac{\cancel{2}t}{\cancel{2}} = \frac{-4}{2}$
$t = -2$